"The Theory of Symmetrical Optical Objectives.—Part II." By S. D. CHALMERS, B.A., St. John's College, Cambridge, M.A. Sydney. Communicated by Professor LARMOR, Sec. R.S. Received January 3,—Read January 26, 1905.

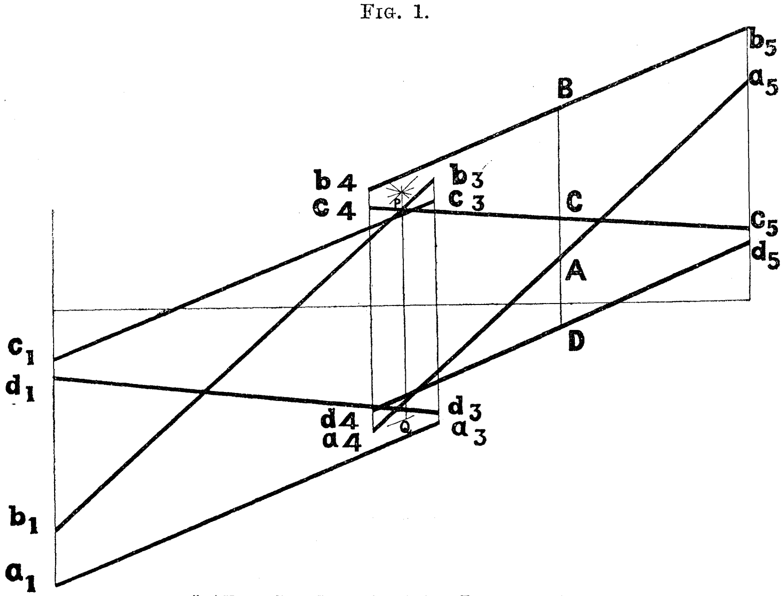
As regards the defects which depend only on those terms in the characteristic function T, that were examined in Part I,* the results of that paper would justify the practice of correcting a single component—the back one—for astigmatism and spherical aberration, provided due attention be paid to the securing, at least approximately, of the condition for no distortion.

But for the values of aperture-ratio and angular field, that obtain in practical systems, the terms of higher orders in T introduce important aberrations, and it is interesting to examine how far the conclusions stated in Part I* are justified in the case of practical systems.

In most cases the discussion can be effected by the use of geometrical relations, it being assumed—as is generally the case in practice—that the stop is well within the focus of the single lens and that its virtual image, formed by either component, is near the actual position of the stop.

In the first place we consider only those rays which lie wholly in one plane. In fig. 1, (5) is the focal plane of the back component,

FIG. 1.



* 'Roy. Soc. Proc.,' vol. 72, June 18, 1903.

(2) the plane of the stop, (3) and (4) its images with respect to the front and back components, and (1) the plane symmetrical to (5); P and Q are two points on the stop equidistant from the centre, a and b two parallel rays through these points, c and d another pair of parallel rays through these points; the intersections of these rays with the planes 1, 3, 4, and 5, being a_1, a_3, a_4, a_5, b_1, etc. The ray d is chosen so that b_4, b_5 is parallel to d_4d_5, then by symmetry it is evident that c_1c_3 is parallel to a_1a_3. It is also evident that the planes 3 and 4, being images of 2, are in every way similar; hence they are the principal planes of the whole system, and the focal plane of the combined system is mid-way between 4 and 5. Let the various rays intersect this plane in A, B, C, and D.

The curvature error* of the combined system can be measured by CA, less the effects due to spherical aberration, where CA evidently

$$= \tfrac{1}{2}\,(c_4a_4 - c_5a_5)$$

but, since b_4b_5 is parallel to d_4d_5,

$$\mathrm{CA} = \tfrac{1}{2}\,\{(b_5a_5 + c_5d_5) - b_4c_4 - a_4d_4)\}.$$

Thus omitting effects of spherical aberration in both cases, it is evident that the curvature error for an angle ω and aperture angle 2α is the mean of the curvature errors for the single system for the angles $(\omega_0 + \frac{1}{2}\alpha)$, and $(\omega_0 - \frac{1}{2}\alpha)$, where $\omega - \omega_0$ represents the angular value of the distortion of the single lens, together with the portion of $\frac{1}{2}\,(b_4c_4 - a_4d_4)$, which is not common to all angles.

When the meridianal astigmatic curve is drawn as usual—the focal lengths being unreduced and the abscissæ representing *angular* field—the ordinates for the combined system will be one-half of those for the single system, subject to the corrections from terms corresponding to $\frac{1}{2}\,(b_4c_4 - a_4d_4)$.

To appreciate the value of the latter expression draw through P the rays a, d, symmetrical to a and d with respect to the axis, then $b_4c_4 - a_4d_4 = c_4a'_4 - c_4d'_4$, and since the angles between b and a, and c and d, are equal, this quantity will, in general, be small when the stop and its image are close together.

To examine the spherical aberration let c_4c_5 be parallel to the axis, a_4a_5 also parallel to the axis, then d_1d_3 and b_1b_3 are also parallel to the axis (fig. 2). The spherical aberration of the whole system is

$$\mathrm{DB} = \tfrac{1}{2}\,(b_5d_5 - b_4d_4);$$

but

$$c_4a_4 = c_5a_5,$$

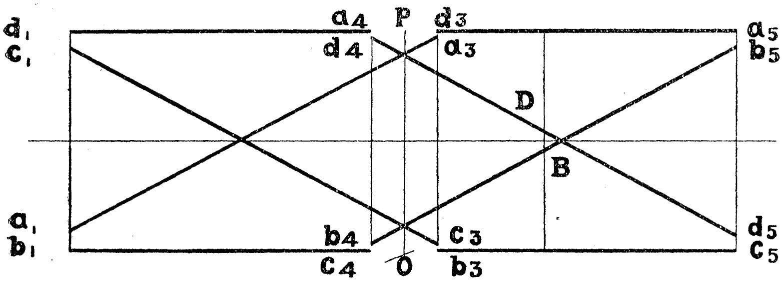
therefore

$$\mathrm{DB} = \tfrac{1}{2}\,\{c_5d_5 + a_5b_5 - (c_4b_4 + a_4d_4)\}.$$

* This term is used for the defect due to the parallel rays through P and Q not intersecting on the focal plane, the effects of spherical aberration being allowed for.

Thus, if the single system be corrected for spherical aberration and curvature, the spherical aberration of the combined system is given by $\frac{1}{2}(c_4b_4+a_4d_4)$. It is evident that this quantity will also be very small.

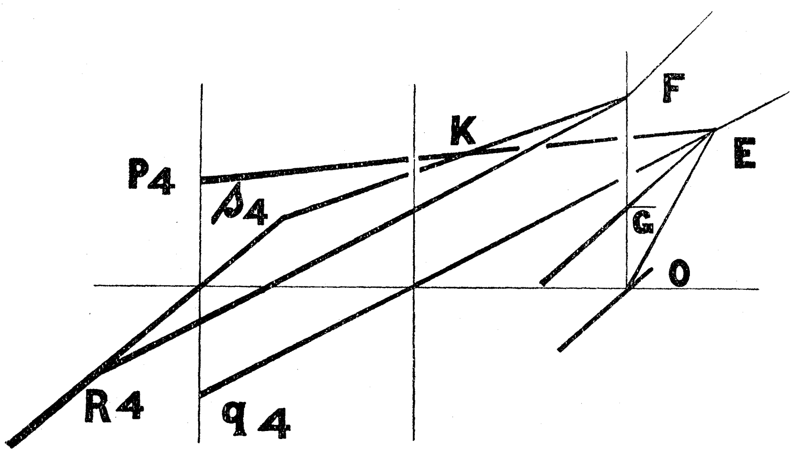
FIG. 2.



These results can be extended to the case of rays which do not lie wholly in one plane. It will be assumed that the image of the stop formed by one member is perfect, the errors thus introduced being of the same nature as those discussed before.

Let PQ and RS (fig. 3) be two diameters of the stop, their image

FIG. 3.



points in the plane 4 being $p_4\ q_4\ r_4\ s_4$; let parallel rays a and b pass through p_4 and q_4 respectively, and let the axial plane parallel to them cut plane 5 in OE; similarly parallel rays c and d pass through r_4 and s_4 and are parallel to the axial plane which cuts 5 in OF.

It being assumed that these pairs of rays intersect on the focal plane 5, the rays a and b will intersect on OE, say at E, and similarly

c and *d* on OF at F. Through E draw EG parallel to RS, cutting OF in G. Now the rays *a b c* and *d* can be chosen so that EG = GF = $\frac{1}{2}$PQ. It is now evident from the figure that Fr_4 is parallel to Eq_4 and that Fs_4 and Ep_4 intersect in *k* on the plane mid-way between 4 and 5, which is the focal plane of the combined system.

But by symmetry the rays *b* and *c* in the second medium correspond to *a* and *d* in the first; hence the parallel rays *a* and *d* intersect on the focal plane of the combined system, but as RS is inclined at any angle to PQ all other rays parallel to *a* and *d* will also intersect at the same point.

Thus we have shown that, subject to the errors introduced by the want of correspondence of the stop and its image, the combined system is completely corrected for astigmatism, curvature of field, and spherical aberration, provided the back component is so corrected. This want of correspondence introduces some slight errors, but in practical systems these are almost negligible.

These conclusions accord very well with the deductions made by Dr. W. Zschokke, from the results of calculating the paths of rays through the various Goerz double symmetrical lenses.